SoPhA 2018: contribution submission

Contribution format: **symposium**

Title:

Continuity, Mathematisation, and Conceptual Change
Continuité, mathématisation et changement conceptuel

Long anonymous abstract describing each contribution :

The main goal of this symposium is to discuss different conceptions of continuity, from Aristotle to the first set-theoretical developments of the notion, both in physics and mathematics. The discussion aims to highlight some aspects of what we take to be the philosophical and scientific depth of the notion of continuity. In particular, the reflexion on continuity will give us the occasion to make two main philosophical points: on the one hand, the mathematisation of the notion of continuity, which made it possible to develop some central empirical concepts in the physics of motion, allows us to argue for the thesis that mathematical language can play a central role in the conceptual elaboration of empirical sciences. On the other hand, the existence of different mathematical notions of continuity offers us direct evidence of cases of incommensurability in mathematics. In this connection, we will discuss the possibility of taking into account the change of meaning of a given term (e.g. 'continuum') in the semantic interpretation of our mathematical statements. The whole discussion will also point out two different lines on which the notion of continuity develops through the history of science: namely, a spatial model and a temporal one.

The symposium's discussion will be conducted as follows: the first talk analyses Aristotle's notion of continuity as a property of both physical and geometrical objects that cannot be understood in set-theoretical terms. The second presentation takes into account the passage from the *geometrisation* to the *mathematisation* of motion, from Galileo to Varignon: in this connection, we will discuss how the mathematisation of the notion of continuity made it possible to develop some central empirical concepts in physics, by focusing on the role of mathematical language within a process of scientific change. Our third talk will analyse Descartes' notion of continuity, highlighting how the theoretical force of the concept of continuity is determinant for the development of the Cartesian philosophical and geometric conception of space. The following presentation will deal with the development of a set-theoretical version of the notion of continuity, showing how Dedekind's and Cantor's constructions of the continuum in arithmetical terms can be seen as directly opposed to an Aristotelian, physical understanding of continuity: here the question will be how the change in the extension of of a given mathematical concept may be accepted, and which semantic account for mathematical terms - if any - may allow for that change to be taken into account.

Finally, the last talk will connect the analysis of the Aristotelian notion of continuity with the reflection of the set-theoretical ones discussed, by pointing out the opposition between a temporal and a spatial model of continuity. Building on Hausdorff's work, we will discuss the role played by this opposition in the set-theoretical genesis of topological spaces.

The five presentations are interconnected and conceived in an interactive, dialogical way; moreover, a rich general discussion session will follow the talks.

i) Aristotelian Continuity

The notion of continuity is of fundamental importance for philosophy and mathematics: its long and complex history starts with Aristotle. While he does not invent the term *to suneches,* he is the one who moves it out of the ontological and cosmological context, and makes of it a technical notion to be used in physical and mathematical inquiry, notably in connection with Zeno's paradoxes of infinite divisibility.

Aristotle's codification of continuity is extremely influent, and sets the terms of the inquiry at least until the Early Modern period. Throughout this long history, however, it has usually been assumed that Aristotle's continuity is nothing else than infinite divisibility – i.e. what we nowadays call 'density'. Because of this, nowadays Aristotle's account of continuity is often treated as at most an historical curiosity, not really relevant to the contemporary debate regarding the modern, post-Dedekindian notion.

This is, I believe, a mistake. A thorough reading of Aristotle's texts (notably the *Physics*), reveals that the Aristotelian account of continuity is much more sophisticated and interesting than it could seem at first sight, and that it cannot be reduced to the mere infinite divisibility.

Aristotle defines the term *to suneches* (the continuous) twice in the *Physics*: while it is the definition in terms of infinite divisibility (*Ph.* VI) that survived through the Middle Ages and the Early Modern period, I believe that the other, more complex formulation (*Ph.* V.3) is the most important one. This obscure definition does not refer directly to the property of continuity, but defines what it means, for an object, to be continuous to another one: understanding this relation provides the means to have an intuitive grasp on the property of continuity, which is never explicitly defined by Aristotle, but only alluded to.

Starting from the definition of the relation given in *Ph.* V.3, however, it is possible to reconstruct the missing definition of the property of continuity: I will present it, and then analyse its characteristics, that go beyond the mere infinite divisibility. In such a way, it is possible to recover and understand the full powerfulness of Aristotle's account, and to evaluate properly its relation to the contemporary theory of continuity.

This means not only to cast light on the similarities and the invariants, but also to properly appreciate its peculiarities and the differences from the contemporary account. Notably, (i) Aristotle's continuity is primarily a physical notion, and only secondarily a mathematical one; (ii) continuity is a property of objects (be them physical or geometrical), and there is nothing like the abstract object that we now call 'the continuum'; (iii) what is continuous cannot be composed of or reduced to points: Aristotle's understanding of continuity is not set-theoretical.

References

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ii) On the Role of Mathematical Structures in Conceptual Change

There is a common idea that mathematical structures are formal languages, and as such, they are supposed to have some kind of *representational flexibility* and *conceptual neutrality,* this is, they can be applied to different domains of phenomena and they can express pre-established content without affecting it.

In a similar vein, it is widely accepted that mathematics played a central role in the conceptual development of empirical sciences. But this role is far from being fully understood, and it is not clear how it could be explained if we accept the thesis of the conceptual neutrality of mathematical language.

I will argue that there is plenty historical evidence against this last thesis, notably, coming from mathematical physics. As various philosophers and historians of science have shown, there are cases in which the very content of 'empirical concepts' of scientific theories is deeply affected by the mathematical structure involved in the theory (see Toulmin 1953, Guisti 1994, Panza 2002, Blay 1992 or Roux 2010).

I will analyse of one of these cases in order to argue against the conceptual neutrality of mathematical languages. I will explain the passage from the *geometrisation* to the *mathematisation* of motion from Galileo to Varignon, trying to show how the mathematisation of the notion of continuity made it possible to develop some central (empirical) concepts from the physics of motion, notably that of *instantaneous velocity.*

The main point of this analysis is to show how the use of geometrical methods of representation in physics posited serious constraints to conceptual development and conceptual use to physicists, and how some of these problems were solved when geometry was partially substituted by analytical methods of representation thanks to the development of infinitesimal calculus.

References:

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iii) Continuity in Cartesian Thought: Some Considerations About the Identification Between Extension and Matter in Cartesian Science.

In general terms, the notion of continuity is used by Descartes in different parts of his work under different types of analysis, but obeying our own interests, we identify "continuity" with two principles in particular: 1) the identification of the body as a continuous extension and 2) continuity as a geometric property. On the one hand, we have "continuity" as an explanatory condition for physical phenomena and their magnitudes and, on the other hand, we have "continuity" as the property of the geometric space.

If matter, according to Descartes, is identified with the property of extension, then matter is space; therefore, it is possible to show how the relationship between physics and geometry is dependent on the demonstration of continuity as a common property. The purpose of this argument is to denote the role of "continuity" in the explanatory process of Cartesian physics (through its theory of matter) and "continuity" in its conception of geometric space that allows us to create the bridge towards a geometric compression of physics.

The purpose of our presentation is to emphasize a case where the theoretical force of the concept of "continuity" is determinant for the development of a certain philosophical and geometric conception of space, i.e. continuity as a property in the construction of geometrical forms and as a fundamental property of the composition of the physical world, based on the Cartesian mechanistic analysis of extension and matter. In other words, we will defend the idea that continuity is a theoretical key element for the ontology and metaphysics of Descartes, which has a direct and decisive impact on his conception of geometry and on his views about the composition of the material world and its interrelations.

References :

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M. Panza, Rethinking geometrical exactness. *Historia Mathematica, 38,* 2011.

iv) Continuity and Conceptual Change: Which Semantics for Mathematical Terms?

The attention of the philosophical community has frequently been drawn to the fact that cases of conceptual change occur in mathematics (see Gillies 1992). This phenomenon is at work every time that, in the history of the discipline, incompatible descriptions of (what we take to be) the same object arise. However, the possibility of conceptual change (together with the overall attention to the mathematical practice) is often neglected when it comes to formalize the semantics of mathematical theories.

We will show how the analysis of the notion(s) of continuity offers us direct evidence of the existence of incommensurability in mathematics. Our main focus will be on Dedekind's definitions of continuity and the real numbers (Dedekind 1872), and on Cantor's analysis of the continuum in terms of infinite point sets (Cantor 1895 - 97). These constructions of the continuum in arithmetical terms will be seen as directly opposed to an Aristotelian, physical understanding of continuity.

The philosophical challenge posed by our case study lies in the question of how we may accept the possibility of the change of meaning of a given mathematical term (e.g. 'continuum'), or the possibility of a change in the extension of the corresponding concept. In this connection, we will look to the semantics of mathematical terms, and to the ways in which these terms are taken to refer to objects. The problem of constructing a reference theory for mathematical terms that may be adequate to take into account the phenomenon of conceptual change will be approached starting from the assumption that the only epistemic access to mathematical objects that we have is a linguistic access (Azzouni 1997, Shapiro 2011). Buzaglo (2002)'s and Hannes Leitgeb (forthcoming)'s proposals on these issues will be considered.

References :

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S. Shapiro, "Epistemology of Mathematics: What Are the Questions? What Count as Answers?", *The Philosophical Quarterly* 61(242), 2011, pp. 130-150.

v) La continuité, entre modèle spatial et temporel

Cet exposé s'articulera à la fois à celui relatif au continu aristotélicien et à celui développant les perspectives dedekindiennes et cantoriennes. Notre objectif sera de mettre à jour une tension qui parcourt, dès Aristote, les conceptions de la continuité et partage celles-ci en deux modèles distincts : un modèle temporel et un modèle spatial. Cette démarcation entre ces deux lignes de force, et les tentatives de synthèse auxquelles elle a pu donner lieu, scande l'évolution de la notion et innerve également les premiers développements ensemblistes (pour lesquels la question du continu, comme problème de cardinalité mais aussi comme problème d'une analyse fine de la composition du continu linéaire, est particulièrement prégnante). Nous verrons notamment, avec la prise en compte du cas de Hausdorff et de son exploration axiomatique et relationnelle des concepts de temps et d'espace, que cette tension a joué un rôle non négligeable dans la genèse ensembliste des espaces topologiques.

Après un retour sur les outils terminologiques élaborés par Aristote et sur son analyse de l'infini au livre III de la *Physique*, nous problématiserons la conception temporelle de la continuité qui se dégage de ces réflexions et l'opposerons à un modèle spatial (qui contrevient à la perspective du Stagirite). Nous montrerons ensuite que les premiers développements ensemblistes posent un nouveau cadre d'élaboration conceptuelle pour cette problématique classique –cadre qui a joué un rôle moteur dans le développement du vocabulaire et des approches topologiques (les espaces topologiques fournissant précisément un biais d'étude privilégié pour les applications continues).

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