

On an ancient duality between the continuous and the discrete, and its tenacious refusal to go away.

ABSTRACT

Since the dawn of philosophy, the interconnection between the continuous and the discrete plays a central rôle in attempts to understand the ontology of the world, while defying all attempts at consistent formulation. Bearing historiography in mind, I will revisit from this point of view the paradoxes of Zeno. It is commonly thought that Zeno's notorious arguments on motion are flawed. However, if one takes all the extant source-texts [3] into account, it can be shown without too much difficulty that Zeno's reasoning is sound, and coherent throughout *all* of his arguments. Indeed, in order to interpret Zeno correctly, it is essential to understand the underlying unity connecting the arguments on plurality with those on motion, a connection which is, again, attested by ancient testimonies [7]. This unifying structure bears on a correct — Zenonian — interpretation of a simultaneous “division through and through” [1]. Thus it is possible to construct a mathematical representation for Zeno's fundamental argument without any need to *a priori* refute it [4, 2]. Our representation clarifies how and why the motion paradoxes are merely a variant of the plurality argument. It furthermore sheds light on Aristotle's reasons to introduce two infinities into metaphysics [6], as well as on their connection to his principle of contradiction. Aristotle's choices with respect to this are very different from those of Plato; thus our analysis helps to bring out some crucial differences between Plato's and Aristotle's metaphysics [6]. When Aristotle's edifice starts to crumble, the need is felt to replace it by an ontology compatible with the needs of early modern natural philosophy. But in trying to develop it the paradoxicality of the continuous and the discrete arises from its grave, in the guise of Zenonian-style paradoxes in physics and the foundations of mathematics. The different choices made with respect to their solution shed light on the ferocious clash between Newtonians and Leibnizians [8]. An interesting perspective is furthermore that the Zenonian argument could be reformulated in the more general mathematical framework of category theory, where it reappears as a special case of the duality between the continuous and the discrete *sensu* Lawvere [5]. To summarise: those paradoxes are here to stay.

References

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