Simples, Complexes and Extension

1 Introduction

Extended simples have been thoroughly discussed in metaphysics.[1] Recently, unextended complexes have been investigated as well.[2] Despite this attention, I find the characterizations of both hardly satisfactory inasmuch as they rely on a locational notion of extension that is far too simplistic. According to such a notion, being extended boils down to having a mereologically complex exact location. In this paper, I make a detailed plea to introduce a different notion of extension, phrased in terms of measure theory. My proposal, I argue, has significant philosophical payoffs, that extend far beyond the discussion about extended simples and unextended complexes. The focus of the paper is on such notions, yet, I contend, the implications of the arguments contained are significantly broader, especially in the light of the theoretical role that the notion of extension plays in crucial debates in metaphysics.

2 Preliminaries

I work with the classical set-theoretic construction of space. On top of set-theory, I use quasi-classical mereology[3] (with parthood $\sqsubseteq$ as primitive, and $\sqsubseteq$ = proper parthood, $\circ$ = overlap, $F(x, S) = x$ is the fusion of the members of S), and consider a theory of location (with exact location as primitive) with axioms that will permit to associate to every x its exact location $L(x)$. Three different principles of location will interest us, *No-Colocation*, *Arbitrary Partition*, and *Expansivity* as they're called in the literature.

3 Extended Simples and Unextended Complexes Defined

The unanimous agreement in the philosophical literature amounts to the following: mereologically complex regions are extended; anything that is exactly located at an extended region is an extended entity.[4]
Using the usual *Atom* (A) predicate, we can define *Being Extended*$_L$ (E_L) and *Being Unextended*$_L$ ($\neg E_L$) as:

$$E_L(x) \equiv_{df} \neg A(L(x)) \tag{1}$$

[1] Scala (2002), McDaniel (2007a), and Gilmore (2014).
[2] McDaniel, (2007b: 239-242), and Pickup (2016).
[3] Quasi-classical in that I require *distinct* fusion axioms for material objects and spatial regions.
[4] Scala (2002: 394), McDaniel (2007a: 131), Gilmore (2014: 25-26), Simons (2014: 63), and Pickup (2016: 257).

$$\neg E_L(x) \equiv_{df} A(L(x)) \tag{2}$$

"Being Extended$_L$" boils down to having a mereologically complex *location*. An extended simple ES_L is:

$$\begin{aligned} ES_L(x) &\equiv_{df} A(x) \wedge E_L(x) \\ &\equiv_{df} A(x) \wedge \neg A(L(x)) \end{aligned} \tag{3}$$

Unextended complexes are similarly defined:

$$\begin{aligned} UC_L(x) &\equiv_{df} \neg A(x) \wedge \neg E_L(x) \\ &\equiv_{df} \neg A(x) \wedge A(L(x)) \end{aligned} \tag{4}$$

I find the locational notion of extension not entirely satisfactory. Thus, I find the definitions of extended simples and unextended complexes unsatisfactory as well. The locational notion is useless in providing a *measure* of extension. Without recurring to any other primitives, we cannot express, in general, that "x is less extended than y". The same goes "x is n-times less-extended than y".

4 Measuring Extension

I claim that we can do better by emplying Lebesgue-measure theory. The Lebesgue measure μ on $\mathbb{R}^n$ gives us a precise way to talk about the *extension* of any (measurable) set $S \in \mathbb{R}^n$. The *extension* of the set S is just the $\mu(S)$. For particular sets the μ is exactly what we expect: the *length* of a line interval in $\mathbb{R}^1$, the *area* of a plane figure in $\mathbb{R}^2$, and the *volume* of a solid in $\mathbb{R}^3$. This suggests:

$$Ext_\mu(x) = \mu(L(x)) \tag{5}$$

According to the measure-theoretic notion of extension, the extension of a spatial entity is the Lebesgue measure of its exact location. If this is correct, then the following seems natural definitions of Being Extended$_\mu$ and Being Unextended$_\mu$:

$$\begin{aligned} E_\mu(x) &\equiv_{df} Ext_\mu(x) > 0 \\ &\equiv_{df} \mu(L(x)) > 0 \end{aligned} \tag{6}$$

$$\begin{aligned} \neg E_\mu(x) &\equiv_{df} Ext_\mu(x) = 0 \\ &\equiv_{df} \mu(L(x)) = 0 \end{aligned} \tag{7}$$

An extended entity is an entity that has Lebesgue measure $\mu > 0$. I argue that this notion of extension does not suffer from the problems that were afflicting the locational notion.

I have defined two notions of "being extended", and their negations, i.e. locational notions E_L and $\neg E_L$, and measure-theoretic notions E_μ and $\neg E_\mu$. What are the relations between these notions? The crucial result is that (8) below fails:

$$\neg E_\mu(x) \to \neg E_L(x) \tag{8}$$

To appreciate this, consider any finite union U, or any countable union of Lebesgue-measure 0 regions. It turns out that $\mu(U) = 0$. The simplest case would be that of a region r composed of two distinct points p_1 and p_2, $r = p_1 \cup p_2$. We have: $\mu(r) = 0$, and $\neg A(r)$. This provides a counterexample to (8).
The same goes for extension. The case above provides a counterexample to:

$$E_L(x) \to E_\mu(x) \tag{9}$$

5 Extended Simples and Unextended Composites Revised

The measure theoretic notion of extension can be used to provide another characterization of extended simples, namely:

$$\begin{aligned} ES_\mu(x) &\equiv_{df} A(x) \wedge E_\mu(x) \\ &\equiv_{df} A(x) \wedge \mu(L(x)) > 0 \end{aligned} \tag{10}$$

The results of §4 have profound consequences on the debate over extended simples. For the very same arguments establish that:

$$ES_L(x) \to ES_\mu(x) \tag{11}$$

does not hold. However, both extended simples$_L$, and extended simples$_\mu$, violate the same principles of location, namely *Arbitrary Partition*. I will argue that the situation is different when it comes to unextended complexes.

My take on unextended complexes parallels the one for extended simples:

$$\begin{aligned} UC_\mu(x) &\equiv_{df} \neg A(x) \wedge \neg E_\mu(x) \\ &\equiv_{df} \neg A(x) \wedge Ext_\mu(x) = 0 \\ &\equiv_{df} \neg A(x) \wedge \mu(L(x)) = 0 \end{aligned} \tag{12}$$

The point is that the following does not hold:

$$UC_\mu(x) \to UC_L(x) \tag{13}$$

That is to say that a spatial entity can be an unextended complex$_\mu$, without thereby being an unextended complex$_L$. The case of unextended complexes is more interesting than the case of extended simples in this respect. For it turns out that unextended complexes$_L$ and unextended complexes$_\mu$ violate very different principles of location. In fact unextended complexes$_\mu$ do not violate *any* of these principles. This makes a substantive difference when it comes to their metaphysical possibility.

6 The Metaphysical Possibility of Unextended Complexes

Both McDaniel and Pickup considers unextended complexes that are pointy-complexes. These entities provide counterexamples to *No-Colocation* or *Expansivity* . Unextended simples$_\mu$ do not.

What about *their* metaphysical possibility? If the standard construction of space is on the right track, they are *actual*, therefore they are metaphysically possible.

Consider any countable union of regions with $\mu = 0$. Call it r: r is an example of an unextended complex$_\mu$. The existence of r is guaranteed by the existence of Lebesgue measure 0 regions, and mereological fusion axioms.

What about unextended complexes$_\mu$ that are *material objects*? An argument in favor of their metaphysical possibility runs as follows: (i) material objects that are exactly located at regions of Lebesgue measure 0 are metaphysically possible; (ii) mereological fusions of such objects are metaphysically possible; therefore unextended complexes$_\mu$ that are material objects are metaphysically possible. The crux of the argument is premise (i). I shall rest content to point out that the arguments in the literature already *assume* (i). They then go on to claim that *co-located* pointy objects are possible. Insofar as the argument in favor of the possibility of unextended material complexes$_\mu$ is not hostage of the (controversial) possibility of co-location, it is a much stronger argument.

7 Conclusion

Finally, I advance several suggestions as to how different notions of extension relate, first, to one another and, second, to mereological structure.