Title:
Philosophical Consequences from a Synthetic Model of Peirce's Continuum

Anonymous long abstract (maximum 1250 words) – 1248 words:

The goal of this talk will be to present the philosophical consequences of a new Synthetic Model of Charles Sanders Peirce's Continuum. This model was built by the second author. As far as we know this is the first mathematical model that captures the main properties of Peirce's Continuum[1] namely: 1) Inextensibility; 2) Reflexivity; 3) Supermultitudiness and weldedness; 4) Modality and Potentiality; 5) Genericity.

Via this model, we first intend to show the consistency of these properties and how they are actually realized on it.

Quite unexpectedly, the said Model is constructed in the context of ZFC[2], which is the most widespread framework for mathematics. In fact, the said Model uses ZFC to obtain, through an appropriate interpretation of the construction, a Synthetic Model of the Continuum despite the analytical character of ZFC itself.

In short, the Model is built with set theoretic tools, but using MereoTopological concepts and interpretations, together with a key inversion idea of the set-theoretical membership relation.

From this, we will finally point to the fact that there are far reaching implications for the philosophy of mathematics and the foundations of Geometry, as can be seen in examining the above-mentioned properties:

1) Inextensibility:

There are no ultimate elements, no atoms, but monads or parts that can always be further divided. This is different from what happens with the real line since the set of real numbers is constituted by ultimate elements, by points. But in the said model, as Peirce advocated for, it is not possible to define a "smallest" MereoTopological place.

Even if from a set theoretic point of view, we start with something like an element "x" when we explode the Monad of x (in the sense similar to Non-Standard Analysis) and continue to grow our sequence, from the inverse MereoTopological relationship, we cannot reach any first ultimate element.

Among the philosophical consequences, we obtain a detailed mathematical modelization of aspects of the Aristotelian continuum. For many philosophers and mathematicians inspired by this Aristotelian continuum[3], the general and the intensional are prior to the individual and the extensional; which differs from Quine's extensionalism and his

[1] As shown by Havenel and by Zalamea, among others, Peirce's Conceptualization of Continuity evolved during his career.

[2] Zermelo-Fraenkel axioms for set theory.

[3] Let us only mention names such as Brouwer, Weyl, Thom and Grothendieck.

rejection of modality.

2) Reflexivity:

Each part of the Peircean Continuum within the said Model is isomorphic to the full Continuum.

An intuitive idea of that isomorphy would be to say that 1, 2, 3, ... numeral infinite is isomorphic with 2, 3, ... numeral infinite. As can be easily noticed, reflexivity implies inextensivity.

By having reflexivity in our model of the continuum, we have the Kantian-Peircean property that every part of the Continuum has parts of the same kind, which is a strong opposite to the dominant view of the Continuum built out of atoms or numbers, as is the case for Cantor's and Dedekind's real line, for Non-Standard Analysis à la Robinson or for Conway's Surreal numbers.

3) Supermultitudiness and weldedness:

The Peircean Continuum within the said Model is "too big" to be a set in ZFC, it is a proper class. Moreover, between any two comparable "places" of this Continuum, there is a proper class of other places or parts and then more than any "multitude of individuals". It is noticeable that, within NBG[4] we have proper class as an object, which is why Erlich, - who wanted to modify Peirce's continuum to have an actualized version of it using something like Non-Standard Analysis -, prefers NBG over ZFC, but we consider that something conceptually and philosophically essential is lost by Erlich; whereas it is maintained by the said model, namely, to put it simply, its Aristotelian nature.

This captures Peirce's conception that "a continuum is merely a discontinuous series with additional possibilities", and that the multiplicity of a continuum is beyond all degrees of multitude, which avoids Cantor's paradox, since "Multitude implies an independence in the individuals of one another which is not found in the supermultitudinous". All the determinable points on a continuum are of a multiplicity so great that those points cannot be actualized together, since their supermultitudinality involves that they are welded together. We have new conceptual tools for the notion of Indiscernibility, and for the notion of border; and we have a better ground for a theory of signification since, contrary to the nominalist standpoint, the meaning of a general term is not exhausted by its concrete instantiations[5].

4) Modality and Potentiality:

The Peircean Continuum within the said Model contains no actual points or ultimate elements; there are only extended parts, so you can interpret this as saying that the potential points are "welded".

If we think of the ideal, potential points as proper-class limits of the sequences in the said model of the Continuum, this leads us to the conclusion that we can distinguish any two (or any collection of) already constructed points. But, in ZFC, we cannot distinguish all of them at the same time, which mirror Peirce's requisite that: "qualities

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[4] Von Neumann-Bernays-Gödel set theory.

[5] See Havenel, 2008, p. 113.

would form a collection too multitudinous for them to remain distinct" (Peirce); Individuals, then, do not "remain distinct" in the precise sense that they are indiscernible: "there cannot be a distinctive quality for each individual" (Peirce).
In other words, we can specify a place, a "locus", as much as we want without reaching any ultimate element or point. It always remains a potentiality of further determination; always capable for determination. To define a "point", it would require a sequence of length "too big"; which is impossible as an object, or as a set in ZFC.
A continuous series cannot contain actual points, but it contains potential points, it is a potential aggregate of points. "A continuous line contains no points or we must say that the principle of excluded middle does not hold of these points… [since it] only applies to an individual". Among the philosophical consequences, via the difference between an actualized set and an ideal proper class, we can mention here that it is similar to reaching the limit of what we can talk about, which was conceptualized differently by Kant and Wittgenstein.
5) Genericity:
The Peircean Continuum within the said Model is such that any two parts or monads are isomorphic, so there is a deep uniformity between the different "loci" in the line, and we have the homogeneity in the René Thom's sense.
We have a new understanding of the meaning of Individuality and relationships between individuals, and also that the Aristotelian property, which can also be named the Synthetic, rather than Analytic, aspect of the continuum, allows an approach of the Continuum – Discrete debate via Semantic rather than Ontology. This means that the Synthetic characteristic does not come from the ontological level (starting from continuum rather than elements as for Aristotle & Thom vs Cantor-Dedekind), but from the semantic level, meaning that we do not use set and elements but proper class and an appropriate interpretation of a MereoTopological relationship. This can be understood as a better accomplishment than René Thom's "l'antériorité ontologique du continu sur le discret", showing that the light does not seem to come from Ontology but from showing the possibility, even within the very analytical ZFC, but with a different semantic and interpretation, to demonstrate not only the consistency but the strength of an Aristotelian approach to the continuum – discrete debate.

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