Discreteness as a guiding principle in the search for a theory of quantum gravity

Fundamental discreteness of the gravitational field is predicted in some sense by many approaches to the problem of quantum gravity[1]. Carlo Rovelli and Francesca Vidotto put things the other way around and take this discreteness to be a guiding principle in the search for a quantum theory of gravity, rather than a consequence of their particular (covariant loop) approach. On a certain, fairly standard, reading of Einstein's general relativity the gravitational field is equivalent to spacetime, so when we talk about properties of the gravitational field these will also be properties of spacetime itself. Conversely, the properties of spacetime can be discovered physically by probing the gravitational field. This reading, when taken too literally, raises some tricky conceptual problems which I turn to towards the end of this talk.

One of the aims of quantum gravity research is to unify quantum mechanics, which takes time evolution to be discrete, with general relativity, which deals with a spacetime continuum. Assuming there is some unified fundamental reality, the basic structure of spacetime must be either discrete or continuous; it cannot be both. What reasons do we have for leaning towards fundamental discreteness? Is this just a bias towards quantum mechanics over general relativity?

Rovelli and Vidotto give an alternative motivation for a fundamentally discrete spacetime in the form of a reconstruction of the thought experiment originally put forward by Russian physicist Matvei Bronstein in 1936. This thought experiment demonstrates that, if we take both quantum mechanics and general relativity into account, there is a minimal distance below which measurements of gravitational field variables cannot be made. This is the Planck length. I spell out this reasoning in more detail, and the inevitable conclusion is that we cannot measure arbitrarily short distances in spacetime. The Planck length is significant in setting the scale at which we must consider both general relativistic and quantum theoretic effects, and below which neither can be consistently applied.

Rovelli and Vidotto go considerably further than this, and interpret this minimal distance for measurability of gravitational field variables as a minimal distance simpliciter. Their conclusion is that the whole idea of a spacetime distance breaks down at the Planck length, and so spacetime cannot be divided up infinitesimally, even in theory. A spacetime which is not infinitely divisible is clearly not continuous and so, their argument goes, it must be discrete. This means that the continuous manifolds of general relativity are at best approximations to a discrete underlying structure. The first part of this talk goes through the steps in this reasoning in a fairly non-technical way, focussing on the conceptual issues and noting various gaps and unmotivated assumptions along the way.

My objections to taking Bronstein's thought experiment to be evidence of fundamental discreteness falls into two related categories. Firstly, there is a lack of clarity about the role of measurement in the form of an implicit assumption that what we can measure with our current techniques is all that can possibly exist. Secondly, we are unclear about the nature of the relationship between mathematics and the physical world. This is a general problem in the wider philosophy of physics which is particularly salient in the context of spacetime discreteness. I do not attempt to resolve this

[1] In particular in the two major approaches of String Theory and Loop Quantum Gravity.

huge issue but show how it bears on the current debate. It turns out that there is no necessity in taking spacetime to be a fundamentally discrete structure.

Next I speculate about some of the consequences of this interpretation of Bronstein's thought experiment: how are we to make sense of an underlying discrete structure which is only approximated by a continuous one, when a continuous one is in a certain sense larger or fuller? Naively it would appear as though we gain information through approximation! We are apparently dealing with some non-standard notion of discreteness.

Part of the problem is that discreteness and continuity are usually thought about against a mathematical background spacetime structure, which may or may not be interpreted as being physical. Due to the background independence of general relativity, we are now talking about a structure which is not defined according to some background geometry; we are attempting to describe the background geometry itself. How can we redefine continuity and discreteness in this case?