The Indiscernibility Strategy Against Non-Categoricity

The non-categoricity of a mathematical theory indicates a failure to characterize one structure, that of the standard model (if there is such a model) of the theory. This is often thought to underlie the semantic indeterminacy of the language of the theory. Proving its categoricity, so as to counter the charge of indeterminacy, is typically considered imperative, at least for the mathematical structuralist. Such thoughts motivate, for example, a recent computational structuralist attempt to prove the categoricity of first-order arithmetic.

One can argue, however, that non-categoricity need not raise any philosophical concern for the structuralist. The argument is based on the claim that the non-standard models of a theory (if there are such models) are indiscernible from the standard model. In the case of first-order arithmetic, as Michael Resnik pointed out, indiscernibility would be a consequence of the fact that by applying mathematical induction within the language of the theory one can prove only results that hold for both the standard and the non-standard numbers. Nonstandard numbers, as he put it, track the standard ones in an appropriate way, i.e., by means available within the theory. In other words, all models of firstorder arithmetic are indiscernible within the theory since they are elementary equivalent *modulo* provability: any provable statement is true in all models.

In this paper, I first discuss the notion of model indiscernibility in play here, and then I argue that the strategy suggested by Resnik does not succeed: mathematical structuralism cannot draw support from the indiscernibility of non-isomorphic models, because indiscernibility is not enough to circumvent semantic indeterminacy.